10. Algebra

WHAT IS THE VALUE OF EACH ANIMAL?

The numbers are the sum of the rows and the columns. These are the clues to solve the puzzle.

				13		=	
						=	
				18		=	
10		15	10			=	

To simplify we can substitute each animal with a letter. It is easier and clearer. What is the first data to use? Continue until you find all the hidden values.

Exercise

Put in algebraic language the following sentences or phrases:

a. Five more than 'x'.	
b. A number divided by 3.	
c. Three times a number.	
d. Three times a number minus eight.	
e. Seven years older than Luis' age.	
f. The sum of four times 'x' plus five times 'y'.	

1. ALGEBRAIC LANGUAGE OR SYMBOLIC LANGUAGE

This is the language that expresses numerical relations in which there are variable quantities or unknown quantities.

Because they don't have a fix value; or because it is not known, we use letters to name these.

Formulas and equations are the most important examples.

For example, the area formula of a rectangle: A = b · a

Another example of an equation: two consecutive numbers add up to 15:

x + x +1 = 15

PARTS OF AN ALGEBRAIC EXPRESSION

Variables: They are the letters.

Terms: Each adding part.

Coefficients: The numbers which are multiplying in every term.

Independent term: The number without letter.

Write for $2x^2+3x-4$:

a) Variable:

b) Terms:

c) Coefficients:

2. ALGEBRAIC EXPRESSIONS

MONOMIALS

The elemental algebraic expression consisting of only one term.

Example: $2x^3$

2 is the coefficient, x^3 is the literal part and 5 is the degree.

Similar monomials are those that have the same literal part.

Example

Write three similar monomials to $2x^3$

POLYNOMIALS

These are the sum of monomials of different degree. $2x^2+3x-4$ is a polynomial.

The polynomial degree is the highest degree that it contains.

Example:

What is the degree of that polynomial?

Binomial is a two-term polynomial, trinomial is a three-term polynomial and so on.

Example:

Write one example for each type

NUMERICAL VALUE OF A POLYNOMIAL

This is the value obtained by replacing the variable with a number and performing the operations.

Example:

Write the numerical value for $2x^2 + 3x - 4$

For x = 1

For x = –2

ADDING AND SUBTRACTING MONOMIALS

If the monomials are similar we can add or subtract the coefficients and leave the common literal part.

If the monomials are not similar we get a polynomial.

The opposite of a monomial is the same monomial with the opposite sign.

PRODUCT

We multiply the coefficients and elsewhere the variables. The result is a new monomial whose degree is the sum of the factor degrees.

DIVISION

We divide the coefficients and divide the variables. The result is a new monomial whose degree is the subtraction of the degrees.

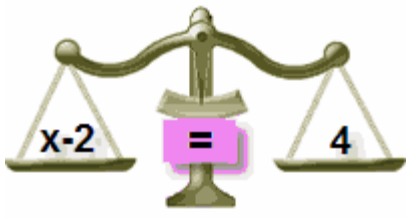
PRODUCT BY A POLYNOMIAL. DISTRIBUTIVE AND COMMON FACTOR

To multiply a monomial by a polynomial we multiply the monomial by each term according to the distributive property.

In the opposite way we can extract a common factor of a polynomial.

3. EQUATIONS

Algebra is the section of mathematics that studies symbolic language.



An equation is an equality with unknowns.

We describe the unknown values by letters.

A solution is any value that makes the equality right.

To solve an equation is the procedure that we follow to find the value of the unknown.

For example,

a) $4x - 5 = 7$; b) $x^2 + x - 2 = 0$; c) $x + y = 7$

Solution is any value that makes the equality right.

Could you write some solution for each previous equation? Check your answers.

a)

b)

c)

To solve an equation is the procedure that we follow to find the value of the unknown. Equivalent equations have the same solutions.

Write an equivalent equation to $3x + 5 = 11$

TRANSPOSITIONS RULES

Two equations are equivalent when they have the same solution.

These are the rules that we use to get equivalent equations. Write on the right the results after using the property. The first one is done as example.

1.	A number which is adding to a member can be removed by subtracting this quantity in both members.	$x + 4 = 7$	$x = 7 - 4$
2.	A number which is subtracting from a member can be removed by adding this quantity in both members.	$3x - 5 = 12$	
3.	A number which is multiplying a member can be removed by dividing this quantity in both members.	$3 \cdot (x - 6) = 9$	
4.	A number which is dividing a member can be removed by multiplying this quantity in both members.	$\frac{5x - 8}{3} = 6$	
5.	We can change the sign of both members together.	$-8x + 5 = 2x - 9$	
6.	If a number is operating in the same way in two members we can simplify it.	a) $5x - \cancel{8} = 6x - \cancel{8}$ b) $(2x + 3) \cdot \cancel{4} = (-2 + x) \cdot \cancel{4}$ c) $\frac{x + 3}{\cancel{7}} = \frac{2x - 9}{\cancel{7}}$	

4. FIRST-DEGREE EQUATION

They are equations whose highest power is one.

RESOLUTION OF EQUATIONS WITH WHOLE COEFFICIENTS

To solve this kind of equation we have to do the following steps:

1. Operating parenthesis.
2. Grouping like terms.
3. Clearing *x*.
4. Checking the solution.

Example:

Solve the following equation: $2 \cdot (3x - 2) - (x + 3) = 8$

RESOLUTION OF EQUATIONS WITH DENOMINATORS

To solve this kind of equations we have to do the following steps:

1. Removing the denominators.
2. Operating parenthesis.
3. Grouping like terms.
4. Clearing x.
5. Checking the solution.

Example:

Solve the equation: $\frac{x}{6}+\frac{2x}{3}=\frac{5}{2}$

5. RESOLUTION OF PROBLEMS WITH EQUATIONS

In science we usually know some information about the unknown values that we want to find. This is the data.

Solving the problem is to find the value of the unknown quantities using the given information.

How to solve them?

You must follow these four steps:

1. Locating the unknown and calling it x.
2. Writing the information in terms of it.
3. Solving the equation.
4. Writing the solution.
5. Checking the solution.

Example:

The sum of three consecutive numbers is 18. Calculate these numbers.

EXERCISES AND PROBLEMS

1. Algebraic language

1. Answer the following questions for the algebraic expression - 4y + 7:

a. Find the numerical value for y = 2

b. Find the numerical value for y = -3

c. Find the numerical value for y = $\frac{5}{2}$

d. Say what its literal term and its constant term are.

2. Answer the following questions for the algebraic expression $\frac{7-2x}{3}$:

a. Find the numerical value for x = –4

b. Find the numerical value for x = 1

c. Find the numerical value for x = 0

d. Find the numerical value for x = –1.

3. Answer the following questions for the algebraic expression 3x – 2:

a. Find the numerical value for x = 5

b. Find the numerical value for x = –2

c. Find the numerical value for x = $\frac{5}{3}$

d. Say what its literal term and its constant term are.

4. Answer the following questions for the algebraic expression –5y + 12:

a. Find the numerical value for y = 2

b. Find the numerical value for y = –3

c. Find the numerical value for y = $\frac{5}{3}$

d. Say what its literal term and its constant term are.

5. Put in algebraic language the following sentences or phrases:

a/ Three consecutive numbers add up to 27

b/ A number and its half add up to 720.

c/ Double of a number is equal to its square.

d/ I ate a quarter of my chocolates and I kept 12 of them. How many chocolates did I have?

6. Complete the table according to the following information:

Jesus is five years older than Pablo. Ramón is three times older than Pablo. Lucia is two years older than Ramón

	Jesus	Pablo	Lucia	Ramón
Four years ago				
Today	x			
Three year later				

2. Algebraic expressions

3. Equations

7. Clear the indicated variable in the following algebraic expressions by applying the transposition rules:

$a)\ m = \frac{a+b}{2}$ clear b $\qquad b)\ F = \frac{9 \cdot C + 160}{5}$ clear C $\qquad c)\ y = \frac{2x+6}{3}$ clear x

4. First-degree equations

8. Solve the next equation and check your answer: 3x – 3(2x – 7) = 4(3x +1) –13

9. Solve the next equation and check your answer: 7x – 4(3x+2) = –8x – 9

10. Solve the equation and check the solution: –4x – 2 (3 + 4x) = –3 (x – 5) + 6

11. Check if x = 0 is a solution for the equation: 2x + 3 = 5.

12. Check if x = 1 is a solution for .the equation: –4x + 6 = –2.

13. Invent an equation with each solution: a) x = 2 b) x = –1 c) x = 3

14. Solve the following equation:
(u + 5) + 2 · (3u – 2) = 8

15. Solve the following equations:

a) $x + \frac{x}{7} = 24$

b) $4x + \frac{1}{2}x = 27$

16. Solve the equation: $\frac{-3+2x}{3} - 2 = \frac{-x+2}{2} + \frac{3x}{5}$

17. Solve the equation: $\frac{5x-4}{6} + 2 = 2x - \frac{7x+1}{8}$

18. Solve the equation: $\frac{-3x+4}{7} - \frac{x+2}{3} = -x + \frac{3}{4}$

19. Solve the following equation: $2(x+1) - 3(x-2) = x + 6$

20. Solve the following equation: $\frac{x-1}{2} - \frac{x+1}{3} = x - \frac{5}{2}$

21. Solve the following equation: $\frac{x+2}{3} - \frac{3x-4}{4} = 2x - 8$

22. Solve the following equations and check the solutions in a) and b):

a) $2(x+5) = \frac{2}{5}(x-3)$ b) $\frac{x+1}{5} = \frac{3x-9}{3}$

$c)\ \frac{21}{x} = 3 \qquad d)\ \frac{9}{x} = -3 \qquad e)\ -2 = 1 + \frac{3}{x} \qquad f)\ \frac{5}{x-1} = \frac{10}{x}$

23. Do the subtraction or addition and simplify: a) $\frac{4(x-3)}{6}-\frac{3x-7}{12}$ b) $\frac{4(x-3)}{6}+2x$

5. Problems

24. The sum of the three consecutive door numbers in a street is 132. What are the numbers? (Remember the consecutive door numbers in a street have an increase of 2 units)

25. Manolo has got a 2 in the first exam and a 6 in the second. What mark does he have to get to pass the subject in the third exam?

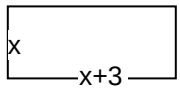
26. The ticket to the swimming pool costs double for an adult than for a boy. A family composed of parents and three siblings pay €24'50. How much is the price of each ticket?

27. I have €5'20 in my pocket. I have coins of 5 cents, 10 cents and 50 cents. How many coins do I have if the number of each type is the same?

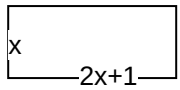
28. I spent half of my money in the cinema and one third on a sandwich. If I kept €2, how much did I have? Write the equation and solve it.

29. I went to school talking to Luis half of the way, then Mary joined us and we were walking together a quarter of the way. Finally, the whole class walked the last 200m. What is the distance from my house to the school?

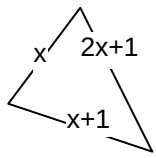
30. The perimeter of this rectangle is 26. How long are its sides?



31. The perimeter of this rectangle is 62cm. How long are its sides?

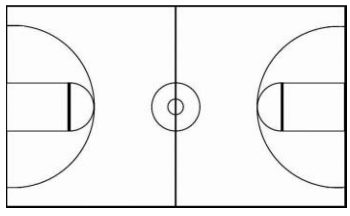


32. The perimeter of this triangle is 62cm. How long are its sides?



33. The three angles of a triangle are consecutive numbers. Write an equation and find each value.

34. We know a basketball field is three times longer than wider. If the perimeter is 52 m, what are the length and the width?



35. Antonio is 7, his brother Richard is 9 and his sister Mary is 10. If the father is 44, how many years have to pass before the father's age is equal to the sum of his children's age?

36. Antonio is fifteen, his brother Roberto thirteen and their father is forty three. How many years have to pass before the father's age is equal to the sum of his children's age?

37. A bicycle costs 180'96 €. What is the price before VAT (16%)?

38. Argue why a number whose digits are three consecutive numbers, like 345 or 543 and so on, is a multiple of three.

39. Change into Fahrenheit the following Celsius temperatures. You have to look for the formula in your last activity.

20ºC; 40ºC

40. In the opposite way, that is, transform into Celsius:
140ºF; 14ºF

41. The ages of a mother and her son add up to 40 years. After 14 years the mother's age will be triple than the son. What are the ages of each one today?

42. In a coffee bar there are 44 seats. There are chairs –4 legs- and bar stools[1] -3 legs-. If the total numbers of legs is 164, how many chairs and stools are there?

43. A student takes a test with 50 questions. The correct answers are 3 points of value and the wrong answers are 2 negative points of value. If the student got 85 points, how many answers did he get right and how many mistakes?

44. An isosceles triangle has as unequal side 10 cm less than the others sides. If the perimeter is 140 cm, what is the length of each side?

45. A rectangle is 18 cm longer than wider and the perimeter is 100 cm. What are its dimensions?

46. Victoria and Aurora are collectors of stamps. Victoria is proud to have 50 stamps more than Aurora. Aurora is going to give 8 stamps as a gift for Victoria's birthday. Then Victoria will have triple than Aurora. How many stamps does each of them have today?

VIDEOS

2. Abstractness:
http://goo.gl/Xi9sZz
3. The beauty of Algebra
http://goo.gl/2we3iZ
4. Descartes and Cartesian coordinates
http://goo.gl/G0Tj5w
5. Why all the letters in Algebra?
http://goo.gl/XT02Ah
6. What is a variable?
http://goo.gl/u3VRHZ
7. Why aren't we using the multiplication sign?
http://goo.gl/OeokMc

[1] Taburete, banqueta.